HYGRO-THERMALLY CURVATURE-STABLE FREE-LAYER COMPOSITE LAMINATES WITH EXTENSION-TWIST COUPLING

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ABSTRACT

The tensile strength and compressive strength are indispensable indexes during the design process of composite laminates. For hygro-thermally curvature-stable extension-twist coupled laminates, the improved Differential Evolution Algorithm combined with penalty function is adopted to solve the nonlinear strong constraint of free-layer laminates. The synchronous optimization of multiple targets is achieved, which includes extension-twist coupled effect, tensile strength, compressive strength and buckling strength. Results are presented for free-layer graphite/epoxy composites laminates that consist of 1-18 plies. Finally the hygro-thermal effect, extension-twist coupled effect and buckling strength are simulated and verified.

1. INTRODUCTION

Many researchers are devoted to studying adaptive composite structures with different coupled effect, in which the bending-twist coupled adaptive structure has a wide application foreground in wind turbine blades and fixed-wing aircraft wings. For example, the bending-twist coupled structure is applied to the aero elastic tailoring of the wing of the fixed wing aircraft, which can control the aerodynamic elastic distortion of the wing and improve the performance of the aircraft [1].

The design of the bending-twist coupled structure can be achieved based on laminates with the extension-twist coupled effect [2]. In order to solve the warping deformation of laminates during the curing process, the programming search method and Sequential Quadratic Programming (SQP) are used to optimize the hygro-thermally curvature-stable (HTCS) laminates with the maximum bending-twist coupled effect by Haynes [3-6]. This method is mainly applied to the lower number of laminates without considering the strength and stability performance indicators. York [7-9] has found HTCS extension-twist coupled laminates with the plies of 0°, ±45°, ±60°, ±90° through programming search and analyzed the buckling load of laminates. This study is only applicable to the laminates with special lay angles, without considering the strength of laminates. On the basis of York’s work, Li [2] has optimized the HTCS antisymmetric \( A_{B,D} \) laminates by SQP method, then the coupling effect and the buckling load of laminates are realized synchronously. However, this study has not achieved the optimization design of free-layer laminates, also without considering the strength index.

In response to above research deficiencies, the HTCS \( A_{B,D} \) laminates is set as the research object in this paper, not only the coupled effect and buckling load but also the tensile strength and compressive strength are included, and the optimization of HTCS free-layer ASBTDS laminates is achieved. The algorithm used for optimization, which called Differential Evolution_Combined Mutation Strategies and Boundary Handling Scheme (DE_CMSBHS), is an improved Differential Evolution algorithm and efficient global optimization algorithm. Combined with the ability to handle constraints of penalty function [10,11], this algorithm can solve the nonlinear constrained problem of optimization effectively [12].

2. MECHANICAL PROPERTIES OF ASBTDS LAMINATES

2.1. Necessary and sufficient conditions

For \( A_{B,D} \) laminates, which has no coupled effect except the extension-shear coupled effect and shear-bend shear coupled, its stiffness equation can be expressed as

\[
\begin{bmatrix}
N_x \\
N_y \\
N_z \\
M_x \\
M_y \\
M_z
\end{bmatrix}
= \begin{bmatrix}
A_{11} & A_{12} & 0 & 0 & 0 & B_{16} \\
A_{21} & A_{22} & 0 & 0 & 0 & B_{26} \\
0 & 0 & A_{66} & B_{16} & B_{26} & 0 \\
0 & 0 & B_{16} & D_{11} & D_{12} & 0 \\
0 & 0 & B_{26} & D_{21} & D_{22} & 0 \\
B_{16} & B_{26} & 0 & 0 & 0 & D_{66}
\end{bmatrix}
\begin{bmatrix}
e_x \\
e_y \\
e_z \\
g_{xx} \\
g_{yy} \\
g_{zz}
\end{bmatrix}
\]

(1)
The stiffness coefficients of laminates can be expressed by the material constant \( U_i \) (\( i=1,2,...,5 \)) and the geometrical factor \( \xi_i \) (\( i=1,2,...,5 \)), as shown in Eq. (2). 

\[
\begin{align*}
A_i &= H U_i + \xi_1 U_1 + \xi_2 U_2 + \xi_3 U_3 + \xi_4 U_4 + \xi_5 U_5, \\
A_n &= H U_n - \xi_1 U_1 - \xi_2 U_2 - \xi_3 U_3 - \xi_4 U_4 - \xi_5 U_5, \\
A_a &= H (\frac{1}{2} U_1 + \xi_1 U_1 + \xi_2 U_2 + \xi_3 U_3 + \xi_4 U_4 + \xi_5 U_5), \\
A_b &= H (\frac{1}{2} U_1 - \xi_1 U_1 - \xi_2 U_2 - \xi_3 U_3 - \xi_4 U_4 - \xi_5 U_5), \\
A_t &= -H (\frac{1}{2} U_1 + \xi_1 U_1 + \xi_2 U_2 + \xi_3 U_3 + \xi_4 U_4 + \xi_5 U_5), \\
A_0 &= -H (\frac{1}{2} U_1 - \xi_1 U_1 - \xi_2 U_2 - \xi_3 U_3 - \xi_4 U_4 - \xi_5 U_5), \\
D_i &= \frac{H}{12} (U_i + \xi_1 U_1 + \xi_2 U_2 + \xi_3 U_3 + \xi_4 U_4 + \xi_5 U_5), \\
D_n &= \frac{H}{12} (U_n - \xi_1 U_1 - \xi_2 U_2 - \xi_3 U_3 - \xi_4 U_4 - \xi_5 U_5), \\
D_a &= \frac{H}{12} (\frac{1}{2} U_1 + \xi_1 U_1 + \xi_2 U_2 + \xi_3 U_3 + \xi_4 U_4 + \xi_5 U_5), \\
D_b &= \frac{H}{12} (\frac{1}{2} U_1 - \xi_1 U_1 - \xi_2 U_2 - \xi_3 U_3 - \xi_4 U_4 - \xi_5 U_5), \\
D_t &= \frac{H}{12} (\frac{1}{2} U_1 + \xi_1 U_1 + \xi_2 U_2 + \xi_3 U_3 + \xi_4 U_4 + \xi_5 U_5), \\
D_0 &= \frac{H}{12} (\frac{1}{2} U_1 - \xi_1 U_1 - \xi_2 U_2 - \xi_3 U_3 - \xi_4 U_4 - \xi_5 U_5),
\end{align*}
\]

\( (2) \)

Where

\[
\begin{align*}
\xi_1 &= \frac{1}{2} \cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 \cos 2\theta_4 \cos 2\theta_5, \\
\xi_2 &= \frac{1}{2} \cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 \cos 2\theta_4 \sin 2\theta_5, \\
\xi_3 &= \frac{1}{2} \cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 \cos 2\theta_4 \sin 2\theta_5, \\
\xi_4 &= \frac{1}{2} \cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 \cos 2\theta_4 \cos 2\theta_5, \\
\xi_5 &= \frac{1}{2} \cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 \cos 2\theta_4 \sin 2\theta_5. \\
\end{align*}
\]

in which the \( \theta_i \) is the layer angle of the \( k \)-ply of laminates, and the \( z_k \) is the position of the \( k \)-ply lamina in the entire laminates, \( n \) is the number of \( k \)-ply plies, and \( H \) is the entire thickness of laminates, as shown in Fig. 1.

**Fig.1:** Composite laminates system

The thermal (humidity) strain \( \varepsilon_{T(H)} \) and \( \kappa_{T(H)} \) caused by the thermal (humidity) internal forces \( N_{T(H)} \) and \( M_{T(H)} \) can be expressed as

\[
\begin{align*}
E_x^{T(H)} &= \begin{bmatrix} a_{11} & a_{12} & a_{16} & b_{11} & b_{12} & b_{16} \end{bmatrix} \begin{bmatrix} N_x^{T(H)} \\ N_y^{T(H)} \\ M_x^{T(H)} \\ M_y^{T(H)} \\ M_O^{T(H)} \end{bmatrix}, \\
E_y^{T(H)} &= \begin{bmatrix} a_{21} & a_{22} & a_{26} & b_{21} & b_{22} & b_{26} \end{bmatrix} \begin{bmatrix} N_x^{T(H)} \\ N_y^{T(H)} \\ M_x^{T(H)} \\ M_y^{T(H)} \\ M_O^{T(H)} \end{bmatrix}, \\
\gamma_{xy}^{T(H)} &= \begin{bmatrix} a_{16} & a_{26} & a_{66} & b_{16} & b_{26} & b_{66} \end{bmatrix} \begin{bmatrix} N_x^{T(H)} \\ N_y^{T(H)} \\ M_x^{T(H)} \\ M_y^{T(H)} \\ M_O^{T(H)} \end{bmatrix}, \\
\kappa_x^{T(H)} &= \begin{bmatrix} b_{11} & b_{12} & b_{16} & d_{11} & d_{12} & d_{16} \end{bmatrix} \begin{bmatrix} N_x^{T(H)} \\ N_y^{T(H)} \\ M_x^{T(H)} \\ M_y^{T(H)} \\ M_O^{T(H)} \end{bmatrix}, \\
\kappa_y^{T(H)} &= \begin{bmatrix} b_{21} & b_{22} & b_{26} & d_{21} & d_{22} & d_{26} \end{bmatrix} \begin{bmatrix} N_x^{T(H)} \\ N_y^{T(H)} \\ M_x^{T(H)} \\ M_y^{T(H)} \\ M_O^{T(H)} \end{bmatrix}, \\
\kappa_{xy}^{T(H)} &= \begin{bmatrix} b_{61} & b_{62} & b_{66} & d_{61} & d_{62} & d_{66} \end{bmatrix} \begin{bmatrix} N_x^{T(H)} \\ N_y^{T(H)} \\ M_x^{T(H)} \\ M_y^{T(H)} \\ M_O^{T(H)} \end{bmatrix},
\end{align*}
\]

Wherein, the sign T(H) represents a variable related to the thermal (humidity) effect, \( \gamma_{xy}^{T(H)} \) indicates the thermal (humidity) shear strain of \( A_{BBD} \) laminates caused by temperature (humidity) change, \( \kappa_{T(H)} \) and \( \kappa_{T(H)} \) are expressed as the surface curvature and torsion curvature of laminates caused by temperature (humidity) changes, the coefficient of flexibility matrix \( a_{jk} \) and \( d_{ij} (i,j=1,2,6) \) can be obtained by the stiffness matrix, especially the \( b_{16} \) is the parameter to test the extension-twist coupled effect of laminates.

Cross [13] has taken into account the arbitrariness of material properties and derived the necessary and sufficient conditions for HTCS composite laminates, as shown in Eq. (5).

\[
\begin{align*}
\xi_1 &= \xi_3 = \xi_5 = 0 \quad \text{or} \quad \xi_2 = \xi_6 = \xi_7 = 0 \quad \text{or} \quad \xi_8 = 0
\end{align*}
\]

The stiffness coefficients \( A_{16} \), \( A_{26} \), \( B_{11} \), \( B_{15} \), \( B_{22} \), \( B_{66} \), \( D_{16} \), \( D_{26} \) of \( A_{BBD} \) laminates are all zero, thence the necessary and sufficient conditions for \( A_{BBD} \) laminates can be derived from Eq. (3).

\[
\begin{align*}
\xi_3 &= \xi_5 = \xi_6 = \xi_7 = 0, \quad \xi_2 = 0, \quad \xi_8 > 0
\end{align*}
\]

Therefore, the necessary and sufficient conditions for HTCS \( A_{BBD} \) laminates can be expressed as Eq. (7).

\[
\begin{align*}
\xi_1 &= \xi_3 = \xi_5 = \xi_6 = \xi_7 = \xi_2 = 0, \quad \xi_8 > 0
\end{align*}
\]

**2.2. The yield strength of laminates**

For composite laminates, when the external load increases to a certain extent, some lamina will be destroyed first. The damage of one lamina cannot mean the destruction of the entire laminates, but it will reduce the stiffness of the entire laminates. If the load continues to increase, more lamina will be destroyed and the stiffness of laminates will further decrease. Therefore, the first yield load for breakage of first lamina \( (N_{y_{10}}) \) is used to measure the strength of laminates.

According to Hill-Tsai strength theory, when the laminate is subjected to an external load \( N_1 \), its first yield load can be solved by Eq. (8).

\[
\begin{align*}
\sigma_{10}^2 X_{10}^2 = \sigma_{T(H)}^2 X_{T(H)}^2 + \sigma_{C}^2 X_{C}^2 + \frac{Z_{C}^2}{S^2} = 1
\end{align*}
\]

Wherein, \( X_{10} \) and \( Y_{10} \) represent the tensile strength and compressive strength of two main in-plane directions of lamina, tensile strength is used when \( N_1 \) is the pulling force, on the contrary the compressive strength is taken, \( S \) represents the shear strength, the stress \( \sigma_{10} \), \( \sigma_{20} \), \( \tau_{12} \) of each lamina can be expressed as
Where \( T \) is the stress direction transformation matrix of lamina, \( Q \) is the stress-strain relationship matrix of the \( k \)-ply lamina, \( A \) is the stiffness matrix of laminates.

Using programming search method, the destroyed load \( N_i \) of each lamina can be calculated by substituting the paving angle of laminates into Eq. (9). Then compare the results of every lamina to find the smallest \( N_i \), which is the first yield load of laminates \( N_{\text{y}} \), as shown in Eq. (10).

\[
N_{\text{y}} = \frac{[N_i]}{H} \tag{10}
\]

### 2.3. The buckling strength of laminates

The buckling strength of \( A_{s}B_{t}D_{s} \) laminates can be solved by buckling differential equation. The length of the laminates is \( a \), the width is \( b \), the aspect ratio is \( c = a/b \), and the buckling load along the direction of the longitudinal is \( N_x \), as shown in Fig. 2.

\[
\text{Fig.2: Laminates under uniform uniaxial in-plane compression}
\]

Since the buckling situation of laminates is not unitary, a concrete expression of the buckling load of laminates is given by means of simple four-sided supporting as [10]

\[
N_x = \left( \frac{a}{m \pi} \right)^2 \left( T_{33} + \frac{2 T_{12} T_{23} T_{13} - T_{22} T_{13}^2 - T_{11} T_{33}}{T_{11} T_{33}} \right) \tag{11}
\]

with

\[
T_{\text{ii}} = A_i \left( \frac{m \pi}{a} \right)^2 + A_{0i} \left( \frac{m \pi}{b} \right)^2 \\
T_{\text{ij}} = A_i \left( \frac{m \pi}{a} \right)^2 + A_{0i} \left( \frac{m \pi}{b} \right)^2 \\
T_{\text{ji}} = A_i \left( \frac{m \pi}{a} \right)^2 + 3B_{i0} \left( \frac{m \pi}{a} \right)^2 + B_{0i} \left( \frac{m \pi}{b} \right)^2 \\
T_{\text{jj}} = B_i \left( \frac{m \pi}{a} \right)^2 + 3B_{i0} \left( \frac{m \pi}{a} \right)^2 + B_{0i} \left( \frac{m \pi}{b} \right)^2 \\
T_{\text{ii}} = D_i \left( \frac{m \pi}{a} \right)^2 + 2Q_{ij} + 2D_{ij} \left( \frac{m \pi}{b} \right)^2
\]

Wherein, \( m \) and \( l \) represent the half-wave numbers in the \( x \) and \( y \) directions respectively. Since \( m \) and \( l \) are integers and variable, programming search now is used to find the minimum \( N_x \) from a large range of \((1 \leq m \leq 100, 1 \leq l \leq 100)\). In order to compare with the yield strength conveniently, the \( N_i \) is used to indicate the buckling strength of laminates, which can be expressed as

\[
N_x = \frac{[N_i]}{H} \tag{13}
\]

### 3. OPTIMIZED DESIGN OF HTCS \( A_{s}B_{t}D_{s} \) LAMINATES

Considering the construction principle of bend-twist coupled structures, which is based on HTCS \( A_{s}B_{t}D_{s} \) laminates, the moments to be borne for bend-twist coupled structures can be equivalent to the tension and pressure of laminates. Thus the tensile strength, compressive strength and buckling strength are significant design indicators for laminates. Only when overall indicators reach the standard can laminates obtain stable mechanical properties.

Combined with the actual engineering requirements, the following assumptions are taken as an example to optimize and design the HTCS \( A_{s}B_{t}D_{s} \) laminates. Both \( N_x \) and \( N_y \) are no less than 240MPa, the buckling strength is no less than yield strength, the size of laminates is set as \( a=200 \text{mm}, b=20 \text{mm} \). For wings structures, the total thickness of laminates is approximately 1-2mm, therefore the 1-18 layers HTCS \( A_{s}B_{t}D_{s} \) laminates will be studied in this paper, which takes the graphite/epoxy composites. Table 1 shows the material properties of the graphite/epoxy composite laminates.

| Material properties of graphite/epoxy composites | \( E_1 \) | 211.0 GPa | \( E_2 \) | 5.3 GPa | \( G_{12} \) | 2.6 GPa | \( v_{12} \) | 0.25 | \( X_t \) | 1.05 GPa | \( Y_t \) | 0.04 GPa | \( X_c \) | 0.70 Gpa | \( Y_c \) | 0.12 GPa | \( S \) | 0.07 GPa | \( t \) | 0.14 mm | \( a_1 \) | -0.02 \( \mu \) ^ {\circ}C | \( a_2 \) | 22.5 \( \mu \) ^ {\circ}C |
|-----------------------------------------------|---------|----------|---------|----------|----------|----------|---------|---------|----------|---------|----------|---------|----------|---------|----------|---------|---------|---------|

The DE_CMSBHS algorithm combined with the penalty function is used to optimize the paving angle of laminates with maximum extension-twist coupled effect, and the mathematical model of optimization problem can be formulated as Eq.(14).

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Table 2 shows the stacking sequence and material properties of 1-18 plies free-layer graphite/epoxy composite HTCS A8B12D6 laminates. As can be seen from the table: (1) There is no HTCS A8B12D6 laminates for 1-7 plies laminates. (2) For 8-18 plies HTCS A8B12D6 laminates, the tensile strength, compressive strength and buckling strength allsatisfy the assumed operating conditions. (3) When laminates is subject to axial compression, the failure of strength will occurs first rather than destabilizing.

4. SIMULATION VERIFICATION

4.1. Verification of hygro-thermal effect

The finite element method is used to verify the distortion caused by the temperature change of the free-layer HTCS laminates. The hygro-thermal effect is verified by the example of the 8-ply laminates. As can be seen from the table: (1) There is no HTCS A8B12D6 laminates for 1-7 plies laminates. (2) For 8-18 plies HTCS A8B12D6 laminates, the tensile strength, compressive strength and buckling strength allsatisfy the assumed operating conditions. (3) When laminates is subject to axial compression, the failure of strength will occurs first rather than destabilizing.

Table 2: Free-layer HTCS A8B12D6 laminates with maximum extension-twist coupled effect

| Number of plies | Stacking sequence/° | $|b_{10}|/m$ | $N_t/M$ pa | $N_c/M$ pa | $N_t/M$ pa |
|-----------------|---------------------|--------------|-------------|-------------|-------------|
| 1-7             | No feasible solution|              |             |             |             |
| 8               | [88.19/8.35/-35.46/53.32/35.46/-8.35/-88.19]T | 8.62×10^7 | 3.30×10^4 | 367.5×10^4 | 256.3×10^4 |
| 9               | [29.97/-30.18/89.96/-29.34/-89.6/2/30.71/89.67/-29.97/-29.79]T | 2.33×10^9 | 1.52×10^4 | 345.1×10^4 | 249.8×10^4 |
| 10              | [6.94/81.82/-80.63/-58.90/-7.86/35.94/-0.98/71.70/-11.60/-85.30]T | 1.48×10^5 | 4.48×10^4 | 463.5×10^4 | 311.0×10^4 |
| 11              | [-29.25/77.38/44.85/-17.32/34.43/-4.40/-52.56/4.91/84.38/-47.6/-5/29.22]T | 8.94×10^6 | 3.14×10^4 | 366×10^4 | 255×10^4 |
| 12              | [81.29/-0.73/76.42/-80.44/21.86/-19.04/4.22/60.16/-18.77/5.75/-84.02/88.78]T | 3.60×10^6 | 8.37×10^4 | 478.1×10^4 | 321.2×10^4 |
| 13              | [-52.50/47.20/8.67/-71.13/58.62/-16.12/10.03/89.39/85.35/-37.99/12.97/-43.61/55.51]T | 2.10×10^6 | 4.95×10^4 | 341.4×10^4 | 246.5×10^4 |
| 14              | [11.85/-59.62/82.83/41.53/-14.93/0/-84.64/-38.73/18.12/76.37/-2.0/4/62.16/42.11/80.57/-17.74]T | 3.68×10^6 | 6.17×10^4 | 407.6×10^4 | 273.1×10^4 |
| 15              | [70.52/-3.81/87.57/-53.76/37.77/-40.11/28.04/25.47/88.61/45.1/3/-1.48/14.01/40.88/-61.61/-88.07]T | 2.46×10^6 | 1.04×10^5 | 352.1×10^4 | 248.7×10^4 |
| 16              | [73.77/-4.93/-38.33/-23.28/74.79/24.66/24.42/88.77/73.87/-41.1/9/49.45/-8.51/3.65/-81.10/-64.02/28.14]T | 6.27×10^6 | 7.01×10^4 | 374.1×10^4 | 259.4×10^4 |
| 17              | [-28.39/73.66/-75.49/1.56/56.71/36.10/-21.31/81.06/6.73/-82.15/-47.38/27.82/-3.48/-34.45/-73.46/85.70/31.24]T | 1.75×10^6 | 1.20×10^5 | 384.5×10^4 | 263.8×10^4 |
| 18              | [-83.77/1.49/50.27/-85.35/-45.56/25.22/-51.56/-8.65/22.14/-25.34/58.43/51.32/-68.84/-89.96/-27.2/0/33.63/-13.33/88.17]T | 2.06×10^6 | 1.35×10^5 | 376.9×10^4 | 252.5×10^4 |
layer HTCS $A_S B_D S$ laminates. The hygro-thermal effect is verified by the example of the 8-ply and 18-ply laminates in Table 2, and the conclusion of other laminates is the same as that of these laminates.

Based on the finite element software MSC.Patran, the finite element model of the 200mm×20mm is established, and 160 shell units are divided. In order to simulate the displacement boundary condition of the composite laminates, the geometric centre of the finite element model is fixed. The typical temperature difference of the high-temperature curing process is 180$^\circ$C to this finite element model. Then the finite element software MSC.Nastran is used to compute with the linear statics calculation function.

The calculated displacement of two kinds of free-layer HTCS $A_S B_D S$ laminates are shown in Fig. 3. It can be seen from the graph that the shear strain, surface curvature and torsion curvature of these laminates are all zero during the high-temperature curing process, which means two $A_S B_D S$ laminates are both hygro-thermally curvature-stable.

The concrete results of the deformation are shown in Table 3, which exhibits that the results of numerical solutions are in good agreement with the analytic solutions, and the reason for error (within 2%) is that the finite element software is biased when simulating the linear uniform load.

The calculated displacement of two free-layer HTCS $A_S B_D S$ laminates under axial extension force is shown in Fig. 4. The figure illustrates that under the axial extension force, laminates not only have axial distortion, but also twist distortion.

$$\text{Fig.4: Displacement nephogram of two laminates due to extension load}$$

The concrete results of the deformation are shown in Table 3, which exhibits that the results of numerical solutions are in good agreement with the analytic solutions, and the reason for error (within 2%) is that the finite element software is biased when simulating the linear uniform load.

4.3. Verification of buckling strength
Taking the finite element method and the same model, the axial uniform pressure of $q=1\text{MN/m}$ is applied at both ends of laminates to verify the buckling strength. The computed results of the first-order

$$\text{Table 3: Numerical and analytic deformation results of HTCS $A_S B_D S$ laminates}$$

<table>
<thead>
<tr>
<th>Number of plies</th>
<th>$\epsilon_x$ analytic solutions</th>
<th>$\epsilon_x$ numerical solutions</th>
<th>Error</th>
<th>$\kappa_x$ analytic solutions</th>
<th>$\kappa_x$ numerical solutions</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>$1.17\times10^6$</td>
<td>$1.15\times10^6$</td>
<td>1.71%</td>
<td>$-8.62\times10^4$</td>
<td>$-8.56\times10^4$</td>
<td>0.70%</td>
</tr>
<tr>
<td>18</td>
<td>$5.35\times10^5$</td>
<td>$5.28\times10^5$</td>
<td>1.31%</td>
<td>$2.06\times10^4$</td>
<td>$2.02\times10^4$</td>
<td>1.94%</td>
</tr>
</tbody>
</table>
buckling factor and buckling load are shown in Table 4. It can be seen from the table that the numerical solutions of the buckling load are in good agreement with the analytic solutions, and the error is less than 2%, which is mainly due to the way of loading.

5. CONCLUSIONS
The stacking sequence and mechanical properties of free-layer HTCS $\mathbf{A}_{13}\mathbf{B}_{7}\mathbf{D}_{5}$ laminates is studied, and the optimization process takes into account the tensile strength, compressive strength and buckling strength of the laminates. Results are obtained for free-layer graphite/epoxy composites laminates that consist of 1-18 plies, which indicates that: (1) The DE_CMSBHS algorithm combined with the penalty function can quickly converge and efficiently obtain the global optimal solution, which can achieve the optimization of laminates under certain conditions of tensile strength, compressive strength and buckling strength. (2) The optimized laminates are all hygro-thermally curvature-stable and have no hygro-thermal shear deformation. (3) The numerical solutions of coupled effect and buckling load are in good agreement with the analytic solutions. (4) The buckling strength of laminates varies with the dimensional change, and the yield strength failure precedes buckling strength failure for laminates in this paper.

The optimization method based on the DE_CMSBHS algorithm proposed in this paper is also suitable for laminates with other coupled effects. Taking the tensile strength and compressive strength into account, the optimal design of the composite laminates is improved in mechanical properties.

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References:

<table>
<thead>
<tr>
<th>Number of plies</th>
<th>First-order buckling factor</th>
<th>Numerical solutions (N/m)</th>
<th>Analytic solutions (N/m)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>345</td>
<td>$3.40 \times 10^6$</td>
<td>$3.45 \times 10^6$</td>
<td>1.47%</td>
</tr>
<tr>
<td>18</td>
<td>373</td>
<td>$3.68 \times 10^7$</td>
<td>$3.73 \times 10^7$</td>
<td>1.36%</td>
</tr>
</tbody>
</table>

Table 4 Numerical and analytic buckling load of HTCS $\mathbf{A}_{13}\mathbf{B}_{7}\mathbf{D}_{5}$ laminates